2019 James S. Rickards Fall Invitational

Name:	
School:	
Score [.]	

1. _____ Compute $92 \cdot 11 + 2(i^{25} + 7) + 3 - \frac{4}{2}$.

- 2. _____ What is half of the circumference of a circle (in units) that has an area of 100 square units?
- 3. _____ How many ways can nine distinguishable people sit at a circular table, if two of them, Ananya and Sruthi, must sit next to each other because they are best friends?
- 4. _____ When you raise the cube root of the number of sides an octagon has to the fourth power, you get the side length of octagon O. Find the area of octagon O.
- 5. _____ How many days are in 9613 hours? Round your answer to the nearest whole number.
- 6. _____ What is the largest prime factor of 6! + 7! + 8!?
- 7. _____ The equation of line A is y = 6x + 7. Line B is perpendicular to line A, and it passes through the point (6,7). What is the y-intercept of line B?
- 8. _____ Compute 2019 + 2 + 0 + 1 + 9 + 20 + 19.
- 9. _____ What is the positive difference between the 13th and the 15th prime numbers?
- 10. _____ How many trailing zeroes does 99! · 101! have?
- 11. _____ How many space diagonals does an octahedron have?
- 12. _____ What is the maximum possible product of two integers, given that their sum is 2019?
- 13. _____ Evaluate $< 20, -5, 21 > \cdot < 2, 10, 19 > .$
- 14. _____ Compute $\sin(2^{2019}\pi) + \cos(2^{2019}\pi) + \tan(2^{2019}\pi)$.
- 15. _____ If $(-1)^x = y$, and x and y are integers, then what is the maximum possible value of $(-1)^y$?
- 16. _____ Find the area of a square, which has a diagonal with length 10.
- 17. _____ What is the constant term in the expansion of $\left(3x \frac{2}{x^2}\right)^3$?
- 18. _____ Which positive integer is the only number that is one more than a perfect square and one less than a perfect cube?
- 19. _____ Which mathematician proved the fact stated in the previous problem?
- 20. _____ What is the volume of a sphere with radius π ?
- 21. _____ What is the sum of the first three perfect squares above 300?

22. _____ Evaluate det(A), where
$$A = \begin{bmatrix} i & i^2 & i^3 \\ i^4 & i^5 & i^6 \\ i^7 & i^8 & i^9 \end{bmatrix}$$

- 23. _____ What is the greatest common factor of 143, 176, and 1001?
- 24. _____ What is the sum of the prime factors of $2^{10} + 1$?
- 25. _____ For what abscissa is the first derivative of the function $f(x) = -x^2 + 10x + 2019$ neither non-negative nor non-positive?
- 26. _____ When you completely simplify $\sqrt{2019^{2019}}$ to get $a\sqrt{b}$, what is the value of $\log_b a$?
- 27. _____ How many ones does the binary representation of 2019 have?
- 28. _____ Compute $5^3 6^3 + 2^7 6^2$.

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- 29. _____ What is the sum of the square roots of the 8th and 9th triangular numbers? (Express your answer in simplest radical form.)
- 30. _____ Compute $19^3 2019$.
- 31. _____ What is the units digit of $2019^{2019} + 2019! + 20^{20} + 19^{19}$
- 32. _____ Akash really wants some delicious Taco Bell. A full meal at Taco Bell consists of an appetizer, a choice between a burrito or a soft taco, fries, and a drink. Taco Bell offers 3 different appetizers, 4 different types of burritos, 6 different soft tacos, 5 types of fries, and 8 types of drinks. How many full meal combinations are there for Akash, considering that he is vegetarian, and instead only has two choices for burritos and four choices for soft tacos?
- 33. _____ Which number is greater, $e + 2\pi$ or 9?

34. _____ What is the largest integer less than $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}}$?

- 35. _____ How many prime numbers are between 50 and 100, inclusive?
- 36. _____ In a randomly shuffled, regular deck of 52 cards, what is the probability of choosing a card with a prime number on it, but it is not a spade?
- 37. _____ Choose a non-zero, real number. Multiply it by 3 and add 4 to your result. Divide this new number by your original number. Which number(s) could you not have gotten?
- 38. _____ How many ways are there to rearrange the letters in the word TESLA?
- 39. \dots Compute 91 · 43.
- 40. _____ In an isosceles triangle, how many of the following centers lie on its Euler line: incenter, circumcenter, orthocenter, centroid, nine-point center?