QUESTION 1

$$g(x) = \begin{cases} ax + 2b & x \leq 0 \\ x^2 + 3a - b & 0 \leq x \leq 2 \\ 3x - 5 & x > 2 \end{cases}$$

Hint: Continuous means you can draw the function without lifting your pen off the paper.

Given $g(x)$, let:

$$\begin{aligned} A &= \text{given that } g(x) \text{ is a continuous function, solve for a} \\ B &= \text{given that } g(x) \text{ is a continuous function, solve for b} \\ C &= \text{if } f(x+1) = x^2 + 3x + 5, \text{ then let } f(x) = C \\ D &= \text{find the inverse of the function: } f(x) = \frac{-1 + \sqrt{4x - 3}}{2} \end{aligned}$$

Solve for A + B + C - D.

QUESTION 2

You are on an island where there are only two types of people: knights, who always tell the truth, and knaves, who never tell the truth. You meet up with a group of 6 people, Alice, Bob, Cathy, David, Eve, and Fred. They tell you the following:

(1) Alice says, "None of us are knights."
(2) Bob says, "At least 3 of us are knights."
(3) Cathy says, "At most 3 of us are knights."
(4) David says, "Exactly 5 of us are knights."
(5) Eve says, "Exactly 2 of us are knights."
(6) Fred says, "Exactly 1 of us is a knight."

Let **A** equal the number of knights in this group of 6.

You are given two empty containers: Container X holds 5 gallons and container Y holds 7 gallons. A step consists of one of the following operations: (1) Fill a chosen container completely with water. (2) Transfer water from one container to another (until either the first is empty or the second is full, whichever comes first). (3) Empty a chosen container onto the ground. Let **B** equal the minimum number of steps required to have exactly 4 gallons in container Y.

Solve for $A + B$.

QUESTION 3

Each side of a pentagon can be colored with one of 5 different colors. However, no two adjacent sides can share the same color. How many different ways can you color the pentagon? Rotations count as different combinations.

QUESTION 4

Evaluate the following summations.

$$\begin{aligned} A &= \sum_{n=0}^{99} \frac{n+3}{n+1} \\ B &= \sum_{n=0}^{99} \frac{n^2-4}{n+1} \\ C &= \sum_{n=0}^{99} \frac{n^2-5n+3}{n+1} \\ D &= \sum_{n=0}^{99} \frac{6n-n^2-1}{n+1} \end{aligned}$$

Evaluate A + B + C + D.

QUESTION 5

Let:

$$\begin{aligned} A &= 2314_5 \text{ in base } 10 \\ B &= \text{the number of positive roots that } x^3 - 4x^2 + 7x^5 - 9 \text{ has} \\ C &= \text{the eccentricity of a parabola} \\ D &= \text{the sum of the first 7 triangular numbers} \end{aligned}$$

Find $A + B + C + D$.

QUESTION 6

Let:

$$\begin{aligned} A &= \text{the focal radius of the graph of the equation: } x^2 - 10x - 8y + 49 = 0 \\ B &= \text{the major axis of the graph of the equation: } 9x^2 - 144x + 16y^2 - 64y + 496 = 0 \\ C &= \text{the eccentricity of the graph of the equation: } 9x^2 - 72x + 16y^2 - 32y + 16 = 0 \end{aligned}$$

Find A + B + C.

QUESTION 7

A : if $x \neq y$, solve for $\dfrac{x}{y}$ in the following equation: $x^2 - 5xy + 4y^2 = 0$

B : if $x^2 - 3x + 1 = 0$, solve for k in the following equation: $x^4 - kx^2 + 1 = 0$

(hint: for both part A and part B, divide by a certain variable)

Find A + B.

QUESTION 8

A : Mihir puts the equation $f(x) = \dfrac{1}{x-1}$ into the form $Px^2 + Qx + R$. Given that x is a non-real complex number and that $x^3 = 2$, solve for $P + Q + R$.

B : The equation $x^{26} - 4x + 2 = 0$ has roots $a, b, c, d...x, y, z$. Find $a^{26} + b^{26} + c^{26} + ... + y^{26} + z^{26}$.

Find $\dfrac{B}{A}$.

QUESTION 9

Given the following equations:

$$a\sqrt{a} - 10\sqrt{a} - 3 = 0$$
$$a + \frac{1}{a} = x$$

Solve for x.

(Hint: Split the middle term in the first equation and then factor.)

QUESTION 10

Find the sum of the numbers of the statements that are true:

(1) The graph for the equation $y = \log x^2$ is the same as the graph for the equation $y = 2\log x$.

(2) For a non-degenerate conic in general form where the coefficients of the x^2 and y^2 terms are both positive and different, the conic will always be an ellipse.

(3) $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$.

(4) For imaginary values, $\sqrt{a} * \sqrt{b} = \sqrt{ab}$.

(5) $\sqrt{x^2} = \pm x$.

(6) If a $\equiv$ b (mod m) and a $\equiv$ b (mod n), then a $\equiv$ b (mod mn).

QUESTION 11

Let:

$$\begin{aligned} A &= \text{the coefficient of the } x^4 \text{ term in } (x^3+3x^2+3x+1)^5 \\ B &= 33^{16} \pmod{16} \end{aligned}$$

Find A + B.

QUESTION 12

Find the determinant of the following matrix:

$$\begin{bmatrix} 5 & 45 & 27 & 21 \\ 42 & 8 & 20 & 4 \\ 1 & 15 & 9 & 7 \\ 19 & 4 & 11 & 2 \end{bmatrix}$$

QUESTION 13

Given the equation $x^6 - 5x^5 + 9x^4 + 3x^3 + 7x^2 - 2x + 4$, calculate the following:

$$\begin{aligned} A &= \text{the sum of the roots of the equation} \\ B &= \text{the product of the roots of the equation} \\ C &= \text{the sum of the roots taken five at a time} \\ D &= \text{the sum of the reciprocals of the roots} \end{aligned}$$

Solve for (A((D * B) - C)).

QUESTION 14

This problem is best done sequentially.

$$
\begin{aligned}
A &= \log_2 2\sqrt{2} + \log_{16} 64 \\
B &= \sum_{x=0}^{\infty} \frac{2}{A^n} \\
C &= \text{the absolute value of the slope of an asymptote of the following hyperbola: } \frac{x^2}{4} - \frac{y^2}{B^2} = -1 \\
D &= \sqrt{2C + \sqrt{2C + ...}}
\end{aligned}
$$

Find $(2D - 1)^2$.