For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Given
$$f(x) = (1 + 7x^3 + 5x)^2$$
, what is the value of $f(2)$?

- (A) 2025 (B) 2209 (C) 4489 (D) 5625 (E) NOTA
- 2. Rationalize the denominator of $\frac{4-\sqrt{2}}{1+\sqrt{2}}$. (A) $6-5\sqrt{2}$ (B) $5\sqrt{2}-6$ (C) -14 (D) 14 (E) NOTA
- 3. If $36^{t} = 6$ and $100^{s} = 1000$, what is $s \cdot t$? (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) NOTA

4. Let a be the sum of the roots of the quadratic and let b be the product of the roots of a quadratic. Find $a \cdot b$ for $2x^2 - 4x - 21$.

- (A) -69 (B) -42 (C) -21 (D) -13 (E) NOTA
- 5. If $x + \frac{1}{x} = 9$, what is value of $x^3 + \frac{1}{x^3}$? (A) 681 (B) 702 (C) 729 (D) 818 (E) NOTA
- 6. Find the value of r + h + s in the following system of linear equations:

7. Tanmay and Shubham can make a taco in 6 days, Karthik and Shubham can make a taco in 3 days, Karthik and Tanmay can make a taco in 5 days. How many days does it take, when all of them are working together, to make a taco if Tanmay runs away to go tell bad jokes at the end of the first day and never comes back? Express your answer as a common fraction in simplest form.

(A) $\frac{35}{18}$ (B) $\frac{15}{7}$ (C) $\frac{52}{19}$ (D) $\frac{59}{17}$ (E) NOTA

8. Ms. Pickett, Ms. Cross, and Ms. Kurian are all running against each other in an election. The ratio of Pickett voters to Cross voters is the same as the ratio of the Cross voters to Kurian voters. If there are 160 Pickett voters and 40 Kurian voters, how many people voted in total? (Assume that Ms. Pickett, Ms. Cross, and Ms. Kurian were the only three candidates running in this election.)

- (A) 280 (B) 360 (C) 420 (D) 560 (E) NOTA
- 9. In how many distinct ways can the letters of RICKHIGH be arranged such that the C and the K are not next to each other?
 - (A) 2520 (B) 2525 (C) 7540 (D) 7560 (E) NOTA
- 10. Find the value of $f^{-1}(12)$, if f(x) = 4x 16. (A) 32 (B) 24 (C) 11 (D) 7 (E) NOTA

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11. Shrung and Nitish want to seek the eternal truth from Eric the Enlightened One. In order to become one of Eric's disciples, they must complete an admissions test comprising of numerous math problems. Shrung and Nitish are stumped on the following question and need your help to solve it.

"If 10 oshs are equal to 12 hmphs, 16 hmphs are equal to 20 pshs, and 40 pshs are equal to 64 wots, how many oshs are equal to 384 wots?"

What answer must Shrung and Nitish give so they can obtain Eric the Enlightened One's knowledge?(A) 140(B) 150(C) 160(D) 170(E) NOTA

12. Vaneesha is at point V(6, 9) and Sukeerth is at the point S(4, 20). Anthony is at point A(6, 6) and Lillian is at point L(3, 4), what is the y-coordinate of the point of intersection of the lines SV and AL? Express your answer as a common fraction in simplest form.

- (A) $\frac{235}{97}$ (B) $\frac{234}{37}$ (C) $\frac{240}{37}$ (D) $\frac{336}{43}$ (E) NOTA
- 13. How many squares can be found in an 4×4 grid, such that each square's sides fall on the lines of the grid and each corner falls on the intersection of grid lines?
 - (A) 30 (B) 40 (C) 42 (D) 44 (E) NOTA

14. Shubham is busy with writing this test while Tanmay is sleeping. Shubham can write 3 questions per hour when Tanmay is asleep. Since Tanmay is so obnoxious, Shubham can only write 2 questions per hour while Tanmay is awake. If Tanmay is asleep for 4 hours and stays awake for the rest of the time, how many hours will it take for Shubham to write the test comprising of 30 questions?

- (A) 10 (B) 13 (C) 15 (D) 12 (E) NOTA
- 15. Evaluate the following expression: $43_5 \cdot 3_{26} \cdot 83_9 36_8$.(A) 5145(B) 5175(C) 24510(D) 24810(E) NOTA

16. Sruthi is trying to find Ananya to make sure that Ananya does not betray their sisterhood, so she decides to travel across the Tallahessian Plane (identical to the Cartesian Plane). First, she has to go to the Buck Lake River, which is located at y = 20, and then she has to go to Ananya's house. Sruthi is currently located at (35, 9) and Ananya's house is at (23, 15). How long is the shortest route that Sruthi can take?

- (A) $3\sqrt{51}$ (B) $2\sqrt{102}$ (C) $\sqrt{346}$ (D) $\sqrt{173}$ (E) NOTA
- 17. Compute: $100^2 99^2 + 98^2 97^2 \dots + 2^2 1^2$. (Hint: Difference of Squares)(A) 1250(B) 3999(C) 5050(D) 7855(E) NOTA
- 18. Karthik is a detective and is very fond of finding patterns and analyzing them. Karthik wants you to be his sidekick. He knows that the location of the criminal will be related to the next number in the following pattern.

Help Karthik an	Karthik and find the next number in the pattern, and solve the mystery!				
(A) 27	(B) 16	(C) 211	(D) 302	(E) NOTA	

19. The number 1729 is known as Ramanujan's number. It is famous for being the smallest number that can be expressed as the sum of two of a certain kind of number, in two different ways. What is this certain kind of number?
(A) Integers
(B) Cubes
(C) Squares
(D) Triangular Numbers (E) NOTA

20. Calculate the integer value of this expression:

21. Calculate the sum of all integer multiples of seven that are under one hundred thousand.

(A)) 102037755 (B) 204075510	(C) 714274285	(D) 1428528570	(E) NOTA
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22. Tanmay and Shubham met when Tanmay was exactly 2 years old in 2006. If today is Tanmay's birthday and he is exactly 15 years old, how many days has it been since Tanmay and Shubham met? (NOTE: Make sure you account for leap days!)

(A) 4745 (B) 4748 (C) 5475 (D) 5478 (E) NOTA (E)

23. Shubham is known for having a beautiful unibrow. His unibrow is 5 and a half inches long. Karthik's eyebrows are two inches long each. How much more 'eyebrow' does Shubham have than Karthik, in inches?

(1) 5.5 mences $(D) 5.5 mences$ $(C) 1 mences$ $(D) 1.5 mences$ $(D) 1.6 mences$	(A) 3.5 inches	(B) 0.5 inches	(C) 1 inch	(D) 1.5 inches	(E) NOT
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- 24. How many integral perfect squares are below one?
 - (A) ∞ (B) 0 (C) 1 (D) 2 (E) NOTA

25. Tanmay needs to know Mr. Juhasz's favorite angle measure. He is relentless to find this angle measure, but he needs your help. The supplement of the complement of Mr. Juhasz's favorite angle measure is equal to seven times his favorite angle. What is Mr. Juhasz's favorite angle measure?

(A) 18° (B) 30° (C) 15° (D) 105° (E) NOTA

26. Shrung is a hoarder, but not a conventional hoarder. Shrung likes to hoard memes on his laptop. He recently made a list of 100,000 memes that he wants to save on his computer. He knows his computer does not have enough space for all of these memes, so he will print out the rest. His computer has 2.5 gigabytes left, and 1,000 memes take up 1 tenth of a gigabyte. Shrung can print 5 memes on one side of a piece of paper and is printing the memes double sided. How many pages of paper will Shrung have to use to store all of his memes, given he maximizes his computer storage first?

- (A) 7,500 (B) 75,000 (C) 15,000 (D) 1,500 (E) NOTA
- 27. Shubham is 14 years old and a very, very fast swimmer. One of his biggest goals in life is to become an Olympic swimmer. He currently has a time of 60 seconds in the 100 meter butterfly event. In order to qualify for the Olympics, he needs to get a time of 54 seconds. Every year he can decrease his time by 0.75 seconds. How old will Shubham be when he is qualified to go to the Olympics?
 - (A) 19 years old (B) 20 years old (C) 22 years old (D) 28 years old (E) NOTA

28. Nitish loves Art of Problem Solving, also known as AoPS. He wants to buy a 300 page book from this website that costs 15 dollars. Assuming he is only paying for the aforementioned pages, how many cents is he paying per page?

- (A) 7 cents (B) 10 cents (C) 5 cents (D) 20 cents (E) NOTA (E) N
- 29. Find the discriminant of the following function: $f(x) = 5x^2 + 8x 38$. (A) 824 (B) $2\sqrt{206}$ (C) 8 (D) -38 (E) NOTA

30. Congratulations on almost finishing the test! Sagar is VERY tired at this point and wants to go to sleep. He looks over at the clock and decides to go to sleep for 15 minutes. What fraction of a day is 15 minutes?

(A)
$$\frac{1}{4}$$
 (B) $\frac{1}{96}$ (C) $\frac{15}{1440}$ (D) 900 (E) NOTA